Basic symbols and algebra notations

Background mathematics review

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Elementary arithmetic symbols

= Equals

+ Addition or “plus” \[2 + 3 = 5\]

– Subtraction, “minus” or “less” \[3 - 2 = 1\]

\(\times\) or \(\cdot\) Multiplication \[2 \times 3 = 6\]

\(\div\) or \(/\) Division \[6 \div 3 = 2\]

\[
\begin{align*}
\frac{\text{(numerator)}}{\text{(denominator)}} &= \frac{\text{(dividend)}}{\text{(divisor)}} = \text{(quotient)} \quad \frac{6}{3} = 2 \quad \frac{6}{3} = 2 \\
\end{align*}
\]
Relational symbols

\[ \equiv \] “Is equivalent to” \[ x / y \equiv \frac{x}{y} \]

\[ \approx \text{ or } \cong \] “Is approximately equal to” \[ \frac{1}{3} \approx 0.33 \]

\[ \propto \] “Is proportional to” \[ a \times x \propto x \]

\[ > \] “Is greater than” \[ 3 > 2 \]

\[ \geq \] “Is greater than or equal to” \[ 1 + x^2 \geq 1 \]

\[ \leq \] “Is less than” \[ 2 < 3 \]

\[ \lesssim \] “Is less than or equal to” \[ 1 \leq 1 + x^2 \]

\[ \gg \] “Is much greater than” \[ 100 \gg 1 \]

\[ \ll \] “Is much less than” \[ 1 \ll 100 \]
# Greek characters used as symbols

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Conventions for multiplication

For multiplying numbers

We explicitly use the multiplication sign “×”

\[ 2 \times 3 = 6 \]

For multiplying variables

We can use the multiplication sign

But where there is no confusion

We drop it

\[ a \times b = c \]

might be simply replaced by

\[ ab = c \]
Use of parentheses and brackets

When we want to group numbers or variables

We can use parentheses (or brackets)

\[ 2 \times (3 + 4) = 2 \times 7 = 14 \]

For such grouping, we can alternatively use

square brackets

\[ 2 \times [3 + 4] = 2 \times 7 = 14 \]

or curly brackets

\[ 2 \times \{3 + 4\} = 2 \times 7 = 14 \]

When used this way, there is no difference in
the mathematical meaning of these brackets.
Associative property

Operations are associative if it does not matter how we group them

e.g., addition of numbers is associative

$$(a + b) + c = a + (b + c)$$

e.g., multiplication of numbers is associative

$$(a \times b) \times c = a \times (b \times c)$$

But

division of numbers is not associative

$$\frac{8}{4} / 2 = 2 / 2 = 1 \text{ but } 8 / \left( \frac{4}{2} \right) = 8 / 2 = 4$$
Distributive property

Property where terms within parentheses can be “distributed” to remove the parentheses

\[ a \times (b + c) = a \times b + a \times c \]

Here, multiplication is said to be distributive over addition

Many other conceivable operations are not distributive, however

E.g., addition is not distributive over multiplication

\[ 3 + (2 \times 5) = 13 \neq (3 + 2) \times (3 + 5) = 40 \]
Commutative property

Property where the order can be switched round

e.g., addition of numbers is commutative
\[ a + b = b + a \]

e.g., multiplication of numbers is commutative
\[ a \times b = b \times a \]

But

e.g., subtraction is not commutative
\[ 5 - 3 = 2 \neq 3 - 5 = -2 \]

e.g., division is not commutative
\[ 6 / 3 = 2 \neq 3 / 6 = \frac{1}{2} \]
Parentheses and functions

A function is something that relates or “maps”

One set of values
Such as an “input” variable or “argument” $x$

To another set of values
which we could think of as an “output”

For example, the function

$$f(x) = x + \frac{1}{4}$$
Parentheses and functions

Conventionally, we say “f of x” when we read \( f(x) \)

Here obviously \( f(x) \) is not “f times x”

Most commonly

Only parentheses are used around the argument \( x \)
not square [ ] or curly { } brackets
Parentheses and functions

For a few very commonly used functions

Such as the trigonometric functions

The parentheses are optionally omitted when the argument is simple

\[ \sin \theta \text{ instead of } \sin(\theta) \]

Note, incidentally,

\[ \sin(-\theta) = -\sin(\theta) \]
Parentheses and functions

For a few very commonly used functions
  Such as the trigonometric functions
    The parentheses are optionally omitted when the argument is simple
      \( \cos \theta \) instead of \( \cos(\theta) \)

Note, incidentally
  \( \cos(-\theta) = \cos(\theta) \)
Sine, cosine, and tangent

Defined using a right-angled triangle

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

Natural units for angles in mathematics are radians

- \(2\pi\) radians in a circle
- 1 radian \(\approx 57.3\) degrees
### Cosecant, secant, and cotangent

**Cosecant**

\[ \csc \theta = \frac{1}{\sin \theta} = \frac{r}{y} \]

**Secant**

\[ \sec \theta = \frac{1}{\cos \theta} = \frac{r}{x} \]

**Cotangent**

\[ \cot \theta = \frac{1}{\tan \theta} = \frac{x}{y} = \frac{\cos \theta}{\sin \theta} \]
Inverse sine function

The inverse sine function $\sin^{-1}(a)$
or arcsine function

Pronounced “arc-sine”

works backwards to give the angle from the sine value

If $a = \sin \theta$ then

$$\arcsin(a) = \text{asin}(a) = \sin^{-1}(a) = \theta$$

Note $\sin^{-1}(a)$ does not mean $1/\sin(a)$

The “-1” here means “inverse function” not “reciprocal”
$\sin^2 \theta$ and $\cos^2 \theta$ functions

However

$$\sin^2 \theta = \sin \theta \times \sin \theta = (\sin \theta)^2$$

Not

$$\sin (\sin \theta)$$

Similarly $\cos^2 \theta = (\cos \theta)^2$

Only trigonometric functions and their close relatives commonly use this notation